

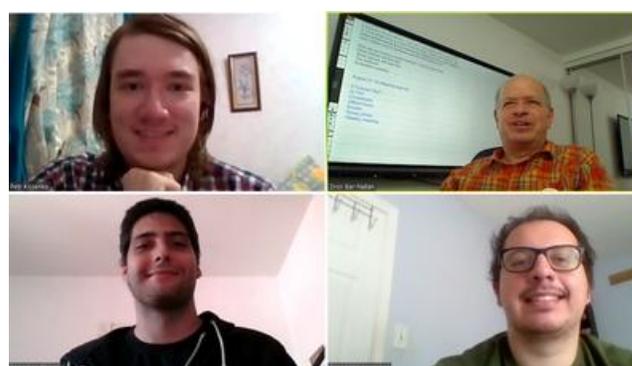
# About This Class

@[Dror Bar-Natan](http://www.math.toronto.edu/~drorbn/) (<http://www.math.toronto.edu/~drorbn/>) @Classes (<http://www.math.toronto.edu/~drorbn/classes/index.html>) @2020-21 (<http://www.math.toronto.edu/~drorbn/classes/#2021>)

**Disclaimers** . This will be a lousy year! We will all work more than in an ordinary year, but achieve less. Expect mishaps! Everything on the MAT 257 web pages remains tentative until it had happened. The Evil Virus situation may force us to make last minute changes to anything.

**Agenda.** Adult-Level Calculus; especially  $\int_M d\omega = \int_{\partial M} \omega$ .

**Instructor.** [Dror Bar-Natan](http://www.math.toronto.edu/~drorbn/) (<http://www.math.toronto.edu/~drorbn/>), [drorbn@math.toronto.edu](mailto:drorbn@math.toronto.edu) (<mailto:drorbn@math.toronto.edu>) (for course administration matters only; math on email is slow and prone to misunderstandings, so I generally avoid it). Office: Bahen 6178 (these days, mostly vacant).



**Teaching Assistants.** Petr Kosenko ([petr.kosenko@mail.utoronto.ca](mailto:petr.kosenko@mail.utoronto.ca) (<mailto:petr.kosenko@mail.utoronto.ca>)) and [Petr's Zoom](https://utoronto.zoom.us/j/95837717811) (<https://utoronto.zoom.us/j/95837717811>), Sebastian Gherghe ([sebastian.gherghe@mail.utoronto.ca](mailto:sebastian.gherghe@mail.utoronto.ca) (<mailto:sebastian.gherghe@mail.utoronto.ca>)) and [Sebastian's Zoom](https://us02web.zoom.us/j/88310099689?pwd=S1dTNmJDZkRBNW5QWENWcWtyWm5QQT09) (<https://us02web.zoom.us/j/88310099689?pwd=S1dTNmJDZkRBNW5QWENWcWtyWm5QQT09>), Panagiotis Angelinos ([peter.angelinos@mail.utoronto.ca](mailto:peter.angelinos@mail.utoronto.ca) (<mailto:peter.angelinos@mail.utoronto.ca>)) and [Peter's Zoom](https://utoronto.zoom.us/j/92163708354) (<https://utoronto.zoom.us/j/92163708354>), and Shuyang Shen ([shuyang.shen@mail.utoronto.ca](mailto:shuyang.shen@mail.utoronto.ca) (<mailto:shuyang.shen@mail.utoronto.ca>)) and [Shuyang's Zoom](https://utoronto.zoom.us/j/83428997680) (<https://utoronto.zoom.us/j/83428997680>) (all zoom passwords are "vchat").



(<http://drorbn.net/AcademicPensieve/Classes/2021-257-AnalysisII/Album>)

Petr Kosenko, Dror Bar-Natan, Sebastian Gherghe, Panagiotis Angelinos, and Shuyang Shen.

**Classes.** Mondays, Wednesdays and Fridays 9-10AM online at <http://drorbn.net/vclass> (<http://drorbn.net/vclass>).

**Office Hours.** With Dror: Tuesdays at 9-10AM and 12-1PM, online at <http://drorbn.net/vchat> (<http://drorbn.net/vchat>) (see [PDF notes](http://drorbn.net/AcademicPensieve/Classes/2021-257-AnalysisII/OfficeHoursNotes.pdf) (<http://drorbn.net/AcademicPensieve/Classes/2021-257-AnalysisII/OfficeHoursNotes.pdf>)), or by appointment (if timing cannot work out, or for issues that

require privacy). Also with Sebastian on Mondays at 11AM-12noon on [Sebastian's Zoom](https://us02web.zoom.us/j/88310099689?pwd=S1dTNmJDZkRBNW5QWENWcWtyWm5QQT09) (<https://us02web.zoom.us/j/88310099689?pwd=S1dTNmJDZkRBNW5QWENWcWtyWm5QQT09>) (password vchat) and with Shuyang on Wednesdays 3-4PM on [Shuyang's Zoom](https://utoronto.zoom.us/j/83428997680) (<https://utoronto.zoom.us/j/83428997680>) (password vchat).

**Online Tutorials** (there will be no in-person tutorials). Wednesdays 5-6PM with Sebastian on [Sebastian's Zoom](https://us02web.zoom.us/j/88310099689?pwd=S1dTNmJDZkRBNW5QWENWcWtyWm5QQT09) (<https://us02web.zoom.us/j/88310099689?pwd=S1dTNmJDZkRBNW5QWENWcWtyWm5QQT09>), Thursdays 4-5PM with Shuyang on [Shuyang's Zoom](https://utoronto.zoom.us/j/83428997680) (<https://utoronto.zoom.us/j/83428997680>) (password vchat), and Fridays 12-1PM with Petr on [Petr's Zoom](https://utoronto.zoom.us/j/95837717811) (<https://utoronto.zoom.us/j/95837717811>) (password vchat).

**Recordings.** For student privacy reasons, you are NOT allowed to record classes, tutorials, and/or office hours. "Official" class recordings showing only the instructors will be made available promptly.

**Text.** Our primary text will be "Little Spivak" - M. Spivak, *Calculus on Manifolds*. Some don't like it because it is very condensed. I like it because it is very condensed. Occasionally we will use material from other sources. Especially from Munkres' *Analysis on Manifolds* and from Folland's *Advanced Calculus*. We may cover the "changes of variables" formula following two short articles by P. Lax, *Changes of Variables in Multiple Integrals, I&II*. A picture of my bookshelf is here too.

**Course Description**, taken from the Faculty of Arts and Science [Calendar](https://fas.calendar.utoronto.ca/section/Mathematics) (<https://fas.calendar.utoronto.ca/section/Mathematics>):

**Hours:** 72L/48T

Topology of  $\mathbb{R}^n$ ; compactness, functions and continuity, extreme value theorem. Derivatives; inverse and implicit function theorems, maxima and minima, Lagrange multipliers. Integration; Fubini's theorem, partitions of unity, change of variables. Differential forms. Manifolds in  $\mathbb{R}^n$ ; integration on manifolds; Stokes' theorem for differential forms and classical versions.

**Prerequisite:** MAT157Y1/MAT157Y5, MAT247H1/MAT247H5.

**Distribution Requirements:** Science

**Breadth Requirements:** The Physical and Mathematical Universes (5)

Note that our choice of material will be a bit different than what is stated in the calendar, and will be closer to what was done in [2016-17](http://drorbn.net/1617-257) (<http://drorbn.net/1617-257>). In this year's unusual circumstances I expect that we will cover less material than in previous years.

**Warning and Recommendation.** This will be a tough and very abstract class, designed for math specialists. I will make every effort to make it understandable, but certain parts of the material require a very high level of mathematical sophistication. Don't take this class unless you are ready to put in the tremendous intellectual effort that will be involved! Every bit of this class absolutely

makes sense. But you'll have to think hard at all times, and be ready to repeatedly adjust your perspective, to see that this is so. Don't let go! If you'll fall behind you'll find it nearly impossible to catch up. This actually does not mean "do your homework in time" (highly recommended anyway). It means "do your deep thinking in time".

**Marking Scheme.** There will be three term test worth 10% each, approximately 20 homework assignments worth a total of 40%, and a final assessment worth all the points you didn't get before, but no more than 50 points (so if, say, you have accumulated 60 points on the term tests and the homework, the final will be worth 40 points, but if you've accumulated only 40, the final will be worth 50 and your maximal possible total will be 90). A renormalization function of the form

$$m \mapsto 100(m/100)^\gamma$$

will then be applied, for a

constant  $\gamma$  which will be chosen later with a bias in favour of  $\gamma = 1$ . Similar renormalizations may or may not also be applied to individual homework assignments or the term tests.

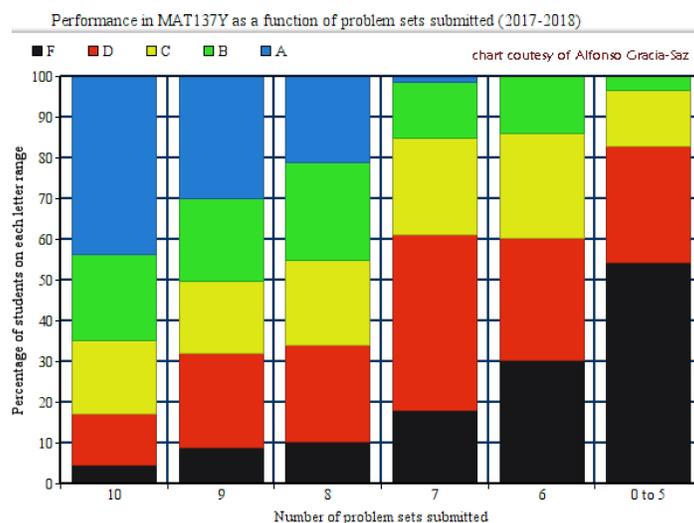
**The Term Tests** will likely take place online at 5-7PM on Tuesday November 3, Tuesday January 19, and Tuesday March 9. A student who misses a term test without providing a valid reason



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within one week of the term test will receive a mark of 0 on that term test. If a student misses the term test for a valid reason (details [here](http://www.illnessverification.utoronto.ca/index.php) [\\_ \(http://www.illnessverification.utoronto.ca/index.php\)](http://www.illnessverification.utoronto.ca/index.php)), the weight of the homework assignments will increase to 44% and the weight the other term tests will increase to 13% each. There will be no make-up term tests.

**Homework.** Assignments will be posted on the course web page (usually on Wednesdays) approximately on the weeks shown in the class timeline. They will be due a week later and they will be (at least partially) marked by the TAs. All students (including those who join the course late) will receive a mark of 0 on each assignment not handed in; though in computing the homework grade, your worst 4 assignments will not count. I encourage you to discuss the assignments with other students or even browse the web, so long as you do at least some of the thinking on your own and you write up your own solutions. Academic integrity rules will not be enforced on homework, so cheating will be easy and may increase your homework grade a bit. But it will hurt your appreciation of yourself, your knowledge, and your exam grades a lot more.



**Solution Sets.** No "official" solution sets for homework assignments and for the term test will be provided. However, I encourage students who got 90% or more on any given assignment (or test) to scan and send me their marked assignments, and I will post their solutions on the class web site as a service to everybody else. Notes:

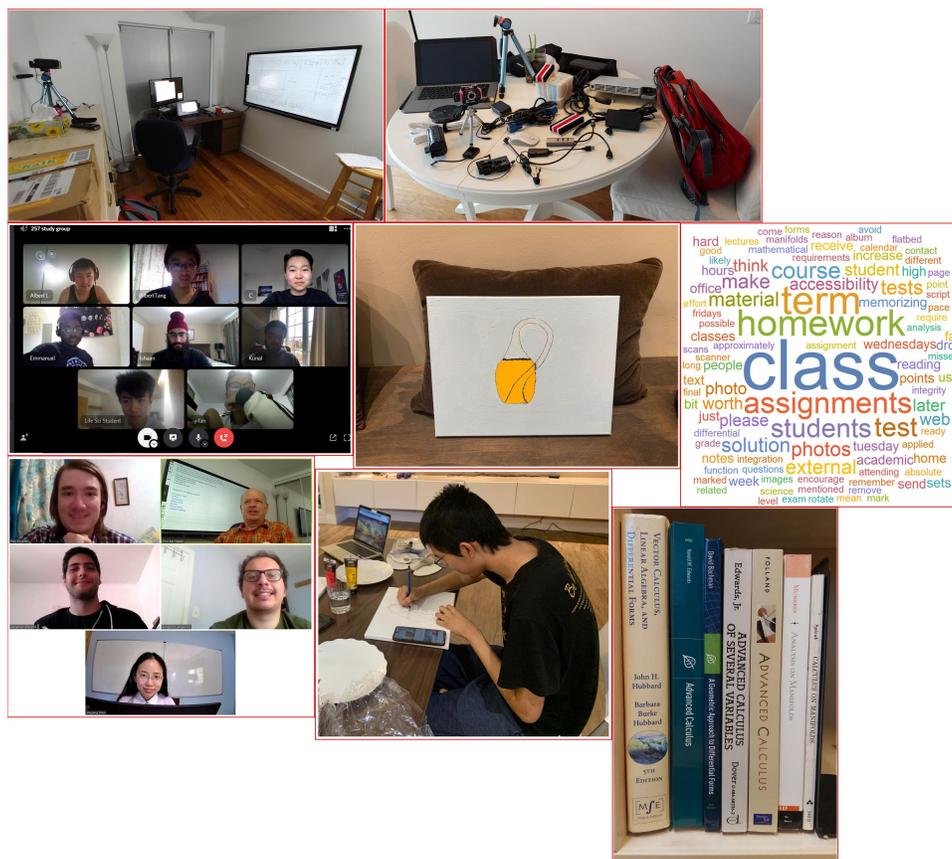
- Please hide student ID numbers in all such scans! You may or may not wish to also suppress your name.
- Scans must be of good quality: they must be of high resolution and contrast, and the paper must look "flat". Use a flatbed scanner or one of those phone apps that simulate a flatbed scanner. Do not use a cellphone camera directly.
- I prefer to receive PDF files, but I'll also take .jpg images if a solution is only 1-3 pages long.
- You may fix and improve your solution set before sending it to me, yet please keep a clear distinction between what was written before submission and what was written after; for

example, use a pen of a different colour for the later edits.

- I will remove all solution sets from the class web site sometime in August 2021.

**Photo Album.** Just for fun, I maintain a public photo album related to MAT257, [here](http://drorbn.net/AcademicPensieve/Classes/2021-257-AnalysisII/Album/) [\(http://drorbn.net/AcademicPensieve/Classes/2021-257-AnalysisII/Album/\)](http://drorbn.net/AcademicPensieve/Classes/2021-257-AnalysisII/Album/)

. Please send me photos! You may send your own portrait or any other photo of you or other students engaging in MAT257-related activities. You may overlay your photos with some caption text, if you wish. If your photos contain images or names of any persons other than yourself you must obtain permission from these people before you submit their photos, and when you submit, you must CC all people seen or mentioned in the photos. If anybody will ever ask me to remove a photo in which they appear or are mentioned, I will do so ASAP with no questions asked.



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**Academic Integrity.** Avoid these troubles! Carefully read the Office of Student Academic Integrity's [Information for Students](http://www.artsci.utoronto.ca/osai/students)  [\(http://www.artsci.utoronto.ca/osai/students\)](http://www.artsci.utoronto.ca/osai/students).

**Cheating** will be possible in MAT257. You are smart and clever and much will be online without watching eyes. If you truly want it and if you don't mind the harm it will do to your knowledge and to your appreciation of yourselves, surely you'll find ways. I'm smart and clever too, and experienced, and my avatars have accounts on several relevant web sites. I will aggressively pursue any cases of suspected cheating while fully respecting all university rules (link here). I will do it out of love and respect, not hate and disgust. I am not disgusted by cheaters; I understand temptation and in the current circumstances I even have sympathy for them. Yet respect for the fair majority will force me to act, and as needed, act I will.

**Telling** - again with love in mind and respect for the fair, I encourage you to tell me if you know

anything. Anonymous is fine and in all cases privacy will be respected. Though keep in mind that I cannot act on words but only on evidence. I care about means even more than I care about names.

**Accessibility Needs.** The University of Toronto is committed to accessibility. If you require accommodations for a disability, or have any accessibility concerns about the course, the classroom or course materials, please contact [Accessibility Services](https://www.studentlife.utoronto.ca/as) (<https://www.studentlife.utoronto.ca/as>) as soon as possible.

### How to Succeed in this Class

- **Keep up!** Don't fall behind on reading, listening, and doing assignments! MAT257 moves at a very high pace. New material is covered once and just once. There will be no going over the same thing again and again - if you fall behind, you stay behind.
- Unless you are an Einstein, there is no way to do well in this class merely by attending lectures - you must think about the material much more than just 3 hours a week if you want it to sink in. And if you are planning on not attending lectures, well, think again. Most people find it very hard to pace their own studies without a human contact; if you'll try, you are likely to discover the hard way that you belong to the majority.
- Take your own class notes, in your own handwriting, and strive to make them as complete as possible. Writing "burns" things into your brain and forces you to keep from daydreaming. And nothing beats reading your own notes when you review the material later on.
- Math is about understanding, not about memorizing. To understand is to internalize; it is to come to the point where whatever the professor does on the blackboard or whatever is printed in the books becomes yours; it is to come to the point where you appreciate why everything is done the way it is done, what does it mean, what are the reasons and motivations and what is it all good for. Don't settle for less!
- This said, you are expected know all definitions and all proofs, and memorizing helps. Memorizing is sometimes the first step towards understanding. If you remember something, you can think about it on the subway ride back home instead of reading advertisements.
- Keep asking yourself questions; many of them will be answered in class, but not all. Remember the old Chinese proverb:

**"Teachers open the door, but you must enter by yourself"**

"师傅领进门,修行靠个人!"